

# An Educational Kit Based on a Modular Silicon Photomultiplier System

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**Abstract**—Silicon Photo-Multipliers (SiPM) are state of the art light detectors with unprecedented single photon sensitivity and photon number resolving capability, representing a breakthrough in several fundamental and applied Science domains. An educational experiment based on a SiPM set-up is proposed in this article, guiding the student towards a comprehensive knowledge of this sensor technology while experiencing the quantum nature of light and exploring the statistical properties of the light pulses emitted by a LED.

**Index Terms**—Silicon Photo-Multipliers, Photon Statistics, Educational Apparatus

## I. INTRODUCTION

**E**XPLORING the quantum nature of phenomena is one of the most exciting experiences a physics student can live. What is being proposed here has to do with light bullets, bunches of photons emitted in a few nanoseconds by an ultra-fast LED and sensed by a state-of-the-art detector, a Silicon Photo-Multiplier (hereafter, SiPM). SiPM can count the number of impacting photons, shot by shot, opening up the possibility to apply basic skills in probability and statistics while playing with light quanta. After an introduction to the SiPM sensor technology (Section II), the basics of the statistical properties of the random process of light emission and the sensor related effects are introduced (Section III). The experimental and data analysis techniques are described in Section IV, while results and discussions are reported in Section V.

## II. COUNTING PHOTONS

SiPMs are cutting edge light detectors essentially consisting of a matrix of photodiodes with a common output, with a density up to  $10^3/mm^2$ . Each diode is operated in a limited Geiger-Muller regime in order to achieve gains at the level of  $\approx 10^6$  and guarantee an extremely high homogeneity in the cell-to-cell response. Subject to the high electric field in the depletion zone, initial charge carriers generated by an absorbed photon or by thermal effects trigger an exponential charge multiplication by impact ionization, till when the current spike across the quenching resistance induces a drop in the operating voltage, stopping the process [1]–[3]. SiPM can be seen as a collection of binary cells, providing

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altogether an information about the intensity of the incoming light by counting the number of fired cells.

Fig. 1 shows the typical response by a SiPM to a light pulse: traces correspond to different numbers of fired cells, proportional to the number of impinging photons. Because of the high gain compared to the noise level, traces are well separated, providing a photon number resolved detection of the light field.

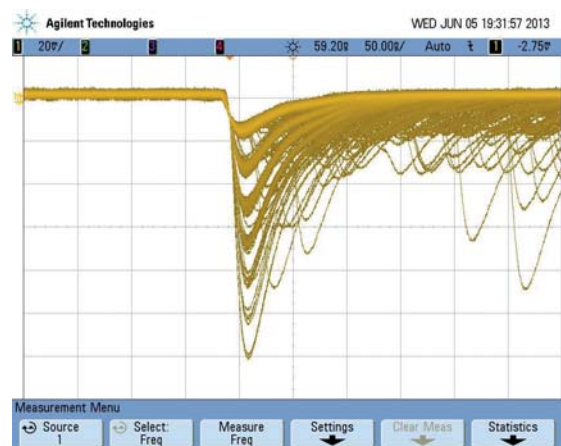


Fig. 1: Response of a SiPM Hamamatsu MPPC S10362-11-100C illuminated by a light pulse.

This is also shown in Fig. 2, displaying the spectrum of the SiPM response to a high statistics of pulses: every entry corresponds to the digitized released charge, measured integrating the electrical current spike during a pre-defined time interval. The peaks correspond to different number of cells fired at the same time. Each peak is well separated and occurs with a probability linked at first order to the light intensity fluctuations. An analysis of the histogram is revealing other significant characteristics:

- The peak at 0 corresponds to no detected photons and its width measures the noise of the system, i.e. the stochastic fluctuations in the output signal in absence of any stimulus. In the displayed histogram,  $\sigma_0 = 29 \pm 1$  ADC channels.
- The peak at 1 detected photon has a width  $\sigma_1 = 38.1 \pm 0.4$  ADC channels, by far exceeding  $\sigma_0$ . The extra contribution may be related to the fact that not all of the cells were born equal. In SiPM the homogeneity of the response is quite high [4], [5], however, since fired

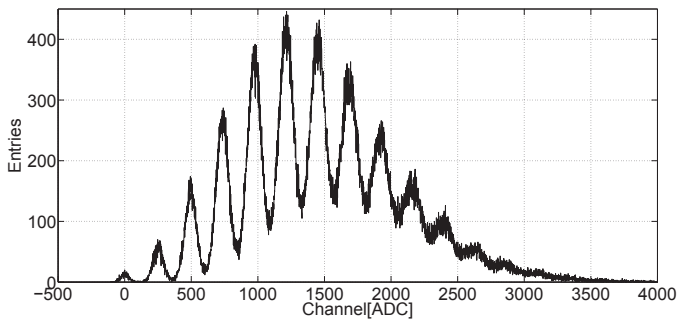


Fig. 2: Photoelectron spectrum probing a LED source measured with a Hamamatsu MPPC S10362-11-100C at a bias voltage of 70.3V and temperature of 25°C.

cells are randomly distributed in the detector sensitive area residuals differences in the gain become evident broadening the peak.

- As a consequence the peak width is increasing with the number  $N$  of fired cells with a growth expected to follow a  $\sqrt{N}$  law, eventually limiting the maximum number  $M$  of resolved peaks.

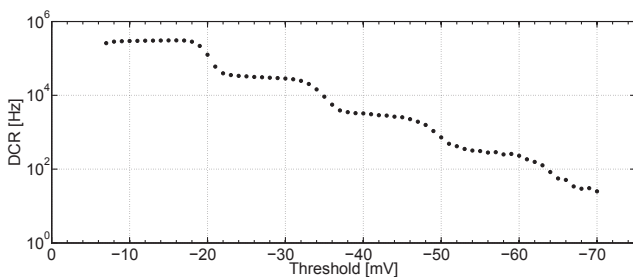


Fig. 3: Measurement of the DCR of the SiPM performed at 25°C.

The detector working conditions can be optimized to maximize  $M$ , properly tuning the bias voltage  $V_{bias}$  and balancing competing effects. On one hand, the peak-to-peak distance is linked to the single cell gain and it is expected to grow linearly with the over-voltage as:

$$Gain = \frac{C \Delta V}{q_e},$$

where  $\Delta V = V_{bias} - V_{Breakdown}$ ,  $C$  is the diode capacitance of the single cell and  $q_e$  the electron charge [2]. Effects broadening the peaks may grow faster dumping the expected resolution. Among these effects it is worth mentioning Dark Count Rate, Optical Cross-Talk and After-Pulsing:

- Free charge carriers may also be thermally generated. Results are spurious avalanches (Dark Counts) occurring randomly and independently from the illumination field. The Dark Count Rate (DCR) does depend from several factors: substrate, processing technology, sensor design and operating temperature [3]. The over-voltage has an impact since the junction thickness volume grows with it

together with the triggering probability, namely the probability that a charge carrier develops an avalanche [3], [4]. The DCR can be measured in different ways. A *Stair Case Plot* is presented in Fig. 3 where the output from a sensor is compared to the threshold of a discriminator and the rate with which the threshold is exceeded is counted. A typical DCR is about  $0.5 \text{ MHz/mm}^2$ .

- Dark Counts may be considered as statistically independent. However, optical photons developed during an avalanche have been shown to trigger secondary avalanches [3] involving more than one cell into spurious pulses. This phenomenon is named Optical Cross-Talk (OCT). The OCT is affected by the sensor technology [3], [6], [7], [10] and strongly depends on the bias voltage increasing the triggering probability and the gain forming the optical photon burst. The OCT can be measured by the ratio of the Dark Counts frequencies for pulses exceeding the 0.5 and 1.5 levels of the single cell amplitude, namely:

$$OCT = \frac{\nu_{1.5pe}}{\nu_{0.5pe}}.$$

The OCT typically ranges between 10% and 20% [4], [10], [11].

- Charge carriers from initial avalanche may be trapped by impurities and released later resulting delayed avalanches, named After-Pulses. For the detectors in use here, an After-Pulse rate at the 7% rate has been reported for an overvoltage  $\Delta V = 1V$ , with a linear dependence on  $V_{bias}$  and a two-component exponential decay time of 15ns and 80ns.

Dark Counts, Optical Cross-Talk and After-Pulses occur stochastically and introduce fluctuations in the multiplication process that contribute to broaden the peaks in the spectrum. An exhaustive study of this effect, also known as Excess Noise Factor (ENF), exceeds the goals of this work and will not be addressed here (see for example [2], [5], [7] and [18]). However, the resolving power that will be introduced in the following may be considered a figure of merit accounting for the ENF and measuring the ability to resolve the number of detected photons.

### III. PHOTON COUNTING STATISTICS

Spontaneous emission of light results from random decays of excited atoms. Occurrences may be considered statistically independent, with a decay probability within a time interval  $\Delta t$  proportional to  $\Delta t$  itself. Being so, the statistics of the number of photons emitted within a finite time interval  $T$  is expected to be Poissonian, namely:

$$P_{n,ph} = \frac{\lambda^n e^{-\lambda}}{n!}, \quad (1)$$

where  $\lambda$  is the mean number of emitted photons.

The detection of the incoming photons has a stochastic nature as well, at the simplest possible order governed by the Photon Detection Probability (PDE)  $\eta$ , resulting in a Binomial probability to detect  $m$  photons out of  $n$ :

$$B_{m,n}(\eta) = \binom{n}{m} \eta^m (1 - \eta)^{n-m}. \quad (2)$$

As a consequence, the distribution  $P_{m,el}$  of the number of detected photons is linked to the distribution  $P_{n,ph}$  of the number of generated photons by

$$P_{m,el} = \sum_{n=m}^{\infty} B_{m,n}(\eta) P_{n,ph}. \quad (3)$$

Nevertheless, it can be demonstrated that the photon statistics is preserved and  $P_{m,el}$  is actually a Poissonian distribution of mean value  $\lambda\eta$  [10], [11]. For the sake of completeness, a less general demonstration is reported in the following.

Multiplying and dividing by  $\eta^n$  each element in the series, Eq. 3 can be written as:

$$\begin{aligned} P_{m,el} &= \sum_{n=m}^{\infty} B_{m,n}(\eta) P_{n,ph}(\lambda) \\ &= \sum_{n=m}^{\infty} \frac{(\lambda\eta)^n \eta^{m-n} (1 - \eta)^{n-m} e^{-\lambda}}{m!(n-m)!}. \end{aligned}$$

Hence, defining  $n - m = z$ :

$$\begin{aligned} P_{m,el} &= \sum_{z=0}^{\infty} (\lambda\eta)^{m+z} \left( \frac{1 - \eta}{\eta} \right)^z \frac{e^{-\lambda}}{m!z!} = \\ &= \frac{(\lambda\eta)^m e^{-\lambda}}{m!} \times \sum_{z=0}^{\infty} \frac{(\lambda\eta)^z}{z!} \left( \frac{1 - \eta}{\eta} \right)^z \\ &= \frac{e^{-\lambda} (\lambda\eta)^m}{m!} \times \sum_{z=0}^{\infty} \frac{(\lambda - \lambda\eta)^z}{z!}. \end{aligned}$$

The series actually corresponds to the Taylor expansion of  $e^{\lambda - \lambda\eta}$ , so that:

$$\begin{aligned} P_{m,el} &= \sum_{n=m}^{\infty} B_{m,n}(\eta) P_{n,ph}(\lambda) = \\ &= \frac{e^{-\lambda\eta} (\lambda\eta)^m}{m!} \end{aligned}$$

Detector effects (especially OCT and After-Pulses) can actually modify the original photo-electron probability density function, leading to significant deviations from a pure Poisson distribution. Following [10] and [11], OCT can be accounted for by a parameter  $\epsilon_{XT}$ , corresponding to the probability of an avalanche to trigger a secondary cell. The probability density function of the number of fired cells can be written at first order as:

$$P \otimes B = \sum_{k=0}^{\text{floor}(m/2)} B_{k,m-k}(\epsilon_{XT}) P_{m-k}(\mu), \quad (4)$$

where  $\text{floor}$  rounds  $m/2$  to the nearest lower integer and  $B_{k,m-k}(\epsilon_{XT})$  is the binomial probability for  $m - k$  cells fired by a photon to generate  $k$  extra hit by OCT.  $P \otimes B$  is characterized by  $\bar{m}_{P \otimes B} = \mu(1 + \epsilon_{XT})$  and  $\sigma_{P \otimes B}^2 = \mu(1 + \epsilon_{XT})$ , where  $\mu = \lambda\eta$ .

#### IV. EXPERIMENTAL TECHNIQUES

In this section the experimental set-up and analysis methods are presented. The optimization of the working point of a SiPM is addressed together with the recorded spectrum analysis technique.

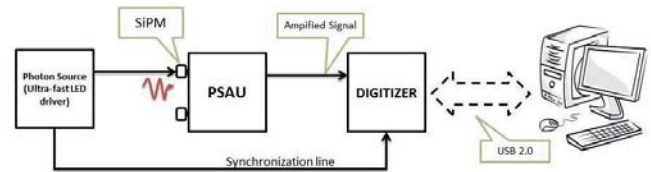


Fig. 4: Schematic layout of the experimental set-up.

##### A. Set-up and measurements

The experimental set-up is based on the CAEN Silicon Photomultiplier Kit. The modular plug and play system contains:

- The Two channel SP5600 CAEN Power Supply and three-stage Amplification Unit [12], with SiPM embedding head unit. The PSAU integrates a leading edge discriminator per channel and coincidence logic.
- The two channels DT5720A CAEN Desktop Digitizer, sampling the signal at 250 MS/s over a 12 bit dynamic range. The available firmware enables the possibility to perform charge Integration (DPP-CI), pulse shape discrimination (DPP-PSD) and advanced triggering [13].
- The ultra-fast LED (SP5601 [14]) driver emitting pulses at 400nm with FWHM of 14nm. Pulses are characterized by an exponential time distribution of the emitted photons with a rising edge at sub-nanosecond level and a trailing edge with  $\tau \approx 5ns$ . The driver is also providing a synchronization signal in NIM standard.

In the current experiments the SiPM that was used is a Multi Pixel Photon Counter (MPPC) S10362-11-100C produced by HAMAMATSU Photonics<sup>1</sup> (see Table I).

The block diagram of the experimental set-up is presented in Fig. 4 with light pulses conveyed to the SiPM by an optical fiber.

The sensor signal is sampled, its area calculated and retained as a figure proportional to the total charge generated

<sup>1</sup><http://www.hamamatsu.com/>.

by the SiPM in response to the impacting photons. The integration window (or *gate*) is adjusted to match the signal development and it is synchronized to the LED driver pulsing frequency. A detailed description of the kit control software and its functionalities is reported in [15].

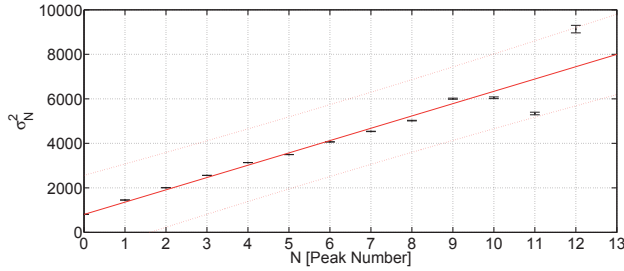


Fig. 5: Peaks width for the spectrum Fig.2 by a multi-Gaussian fit.

The proposed experimental activities start with the optimal setting of the sensor bias voltage, defined maximizing the resolving power defined as:

$$R = \frac{\Delta_{pp}}{\sigma_{gain}}, \quad (5)$$

where  $\Delta_{pp}$  is the peak-to-peak distance in spectrum and  $\sigma_{gain}$  accounts for the single cell gain fluctuations:

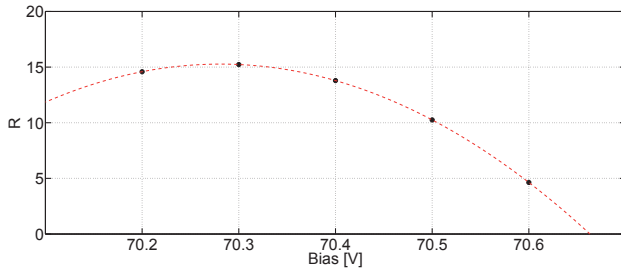


Fig. 6: Scan of the resolution power  $R$  as a function of the bias voltage at fixed temperature ( $25^\circ\text{C}$ ) and light intensity. The working point is given by a polynomial fit and equal to  $70.28\text{ V}$ .

$$\sigma_{gain} = (\sigma_1^2 - \sigma_0^2)^{1/2},$$

being  $\sigma_{0,1}$  the standard deviations of the 0- and 1-photoelectron peaks [16].  $R$  is a figure of merit measuring the capability to resolve neighboring peaks in the spectrum. In fact, following the Sparrow criterion [17] according to which two peaks are no longer resolved as long as the dip half way between them ceases to be visible in the superposed curves, the maximum number  $N_{max}$  of identified peaks is given by:

$$N_{max} < \frac{R^2}{4},$$

where it has been assumed the width of the peaks to grow as the squared root of the number of cells (as confirmed by the data reported in Fig. 5).

A typical plot of the resolving power versus the bias voltage is presented in Fig. 6. The optimal biasing value corresponds to the maximum resolution in the plot and it is used as a working

TABLE I: Main characteristics of the SiPM (Hamamatsu MPPC S10362-11-100C)

Number of Cells:	100
Area:	$1 \times 1\text{ mm}^2$
Diode Dimension:	$100\ \mu\text{m} \times 100\ \mu\text{m}$
Breakdown Voltage:	69.2V
DCR:	540 kHz at 70V
OCT:	25% at 70V
Gain:	$3.3 \times 10^6$ at 70V
PDE ( $\lambda = 440\text{nm}$ ):	65% at 70V

point. After the sensor setting, spectra can be recorded for different light intensities, with a maximum number of fired cells not exceeding  $N_{max}$ .

### B. Multi-peak spectrum analysis

Spectra recorded in response to photons impacting on the sensor can be seen as a superposition of Gaussians, each corresponds to a well defined number of fired cells. The key point in the analysis technique is the estimation of the area underneath every peak, allowing the reconstruction of the probability density functions.

TABLE II: Acquisition parameters for the reference run presented in this work.

$V_{bias}$ [V]	GateWidth [ns]	Trigger frequency [kHz]	Temperature [C]
70.3	300	100	25.0

Initially, areas can be estimated by a *Pick&Play* (hereafter, P&P) procedure on the spectrum. In fact, a binned Gaussian distribution of  $N_{pk}$  events may be written as:

$$y_i = y(x_i) = y_{max} e^{-\frac{(x_i - x_0)^2}{2\sigma^2}},$$

where  $y(x_i)$  is the number of events in the bin centered on  $x_i$  and  $y_{max}$  is the highest bin, in  $x_0$ . Since  $y_{max} = N_{pk}/(\sigma\sqrt{2\pi})$ , knowing the content of the highest bin and estimating  $\sigma$  leads to  $N_{pk}$ . The standard deviation can also be calculated in a simple way by the Full Width at Half Maximum (*FWHM*), obtained searching for the position of the bins with a content equals to  $y_{max}/2$  and presuming that  $FWHM = 2.355 \times \sigma$ . Advantages and limitations of this method are quite obvious: its applicability is straightforward and essentially requires no tool beyond a Graphical User's Interface (*GUI*) for the control of the set-up; on the other hand, it can be applied only to peaks with a limited overlap and uncertainties can only be obtained by repeating the experiment. In order to overcome these limitations, a *Multi-Gaussian Fit* (MGF) procedure was implemented in MATLAB to analyze the full spectrum, according to the following work flow:

- **Initialization.** Robustness and efficiency of minimization algorithms is guaranteed by having an educated guess of the parameter values and by defining boundaries in the parameter variation, a procedure increasingly important as the number of parameters grow. Initial values are provided in an iterative procedure:



- the user is required to identify by pointing & clicking on the spectrum the peak values and their position for 3 neighboring Gaussians, fitted to improve the estimate.
- Initial values for every Gaussian are estimated by relying on the peak-to-peak distance from the previous step, presuming the signal from the 0-cell peak to be centered in the origin of the horizontal scale and assuming the standard deviation grows as the squared root of the number of cells.
- **Fit.** Spectra fitted to a superposition of Gaussians with a non-linear  $\chi^2$  minimization algorithm presuming binomial errors in the content of every bin. The most robust convergence over a large number of tests and conditions have been empirically found bounding parameters to vary within 20% of the initial value for the peak position, 30% for the area and 50% for the standard deviation.

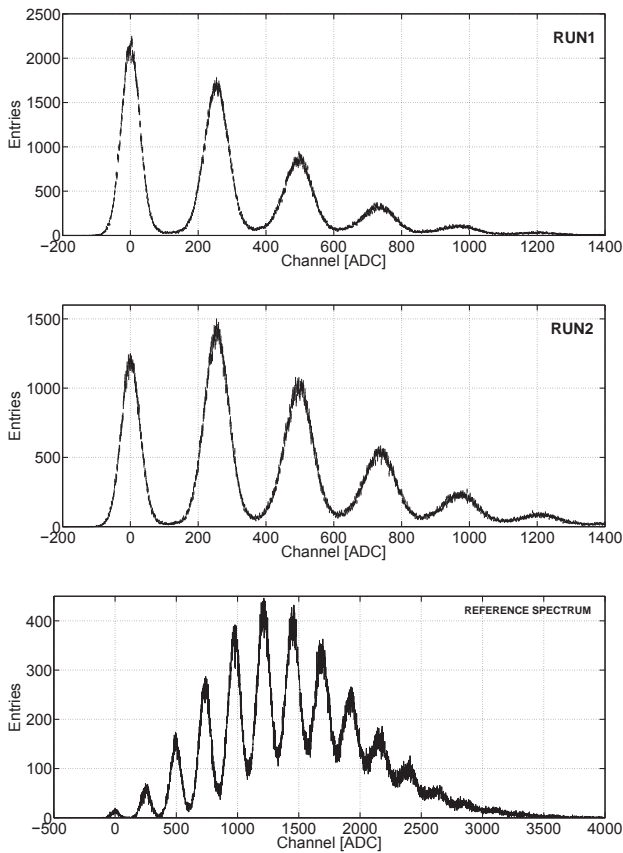


Fig. 7: Exemplary spectra. The mean number  $\mu_{MI}$  of photo-electrons is measured to be  $1.209 \pm 0.002$  (RUN1),  $1.762 \pm 0.002$  (RUN2) and  $6.24 \pm 0.01$  (Reference Spectrum).

## V. RESULTS AND DISCUSSIONS.

Exemplary spectra for three light intensities were recorded and the raw data distributions are shown in Fig. 7, where the horizontal scale in ADC channels measures the integrated charge in a pre-defined gate. In the following, the analysis steps are detailed for the distribution corresponding to the highest mean photon number, hereafter identified as the *Reference Spectrum*. Remaining spectra will be used to assess the

robustness of the approach and the validity of the model, with the results summarized at the end of the section.

Spectra are seen as a superposition of Gaussians, with parameters estimated according to the methods introduced in Section IV. The outcome of the procedures for the *Reference Spectrum* is reported in Table III for the P&P and the MGF procedures. For the former, uncertainties in the estimated parameters are the standard deviations from five data sets acquired in identical conditions while for the latter errors result from the fitting procedure (Fig. 8).

TABLE III: Peak position, width and experimental probability of having N photo-electrons from the Pick&Play (P&P) procedure, compared to the results from the Multi-Gaussian Fit (MGF). The reported results are for the reference spectrum.

N	PeakPosition[ADC]		PeakWidth[ADC]		Exp. Probability	
	P&P	MGF	P&P	MGF	P&P	MGF
0	$3 \pm 1$	$2 \pm 1$	$28.2 \pm 2$	$29 \pm 1$	$0.22 \pm 0.02$	$0.22 \pm 0.01$
1	$257 \pm 2$	$253.0 \pm 0.6$	$39 \pm 2$	$38.1 \pm 0.4$	$1.41 \pm 0.09$	$1.32 \pm 0.02$
2	$498 \pm 1$	$496.1 \pm 0.4$	$41.7 \pm 0.5$	$44.8 \pm 0.3$	$4.3 \pm 0.1$	$4.16 \pm 0.03$
3	$737 \pm 2$	$735.5 \pm 0.3$	$47.6 \pm 0.3$	$50.6 \pm 0.3$	$8.2 \pm 0.2$	$8.29 \pm 0.05$
4	$978 \pm 1$	$974.8 \pm 0.3$	$54.4 \pm 0.3$	$55.9 \pm 0.3$	$12.3 \pm 0.6$	$12.62 \pm 0.06$
5	$1215 \pm 2$	$1212.8 \pm 0.3$	$58 \pm 1$	$59.1 \pm 0.4$	$15.1 \pm 0.6$	$15.23 \pm 0.08$
6	$1454 \pm 2$	$1450.3 \pm 0.4$	$66.2 \pm 0.9$	$63.8 \pm 0.5$	$16.4 \pm 0.6$	$15.6 \pm 0.1$
7	$1689 \pm 2$	$1686.2 \pm 0.6$	$75 \pm 2$	$67.4 \pm 0.7$	$14.7 \pm 0.6$	$13.6 \pm 0.1$
8	---	$1919.2 \pm 0.8$	---	$71 \pm 1$	---	$10.5 \pm 0.2$
9	---	$2155 \pm 1$	---	$73 \pm 5$	---	$7.6 \pm 0.2$
10	---	$2393 \pm 2$	---	$77 \pm 2$	---	$4.7 \pm 0.2$
11	---	$2618 \pm 3$	---	$78 \pm 3$	---	$2.3 \pm 0.2$
12	---	$2830 \pm 9$	---	$96 \pm 12$	---	$1.4 \pm 0.3$
13	---	$3083 \pm 34$	---	$170 \pm 20$	---	$1.2 \pm 0.2$

The characteristics of the experimental distribution can initially be studied referring to the mean number of fired cells. A *Model Independent* (MI) estimate is provided by

$$\mu_{MI} = \frac{\overline{ADC}}{\Delta_{pp}}, \quad (6)$$

where

$$\overline{ADC} = \frac{\sum_i y_i ADC_i}{\sum_i y_i} \quad (7)$$

is the mean value of the experimental distribution (being  $y_i$  the number of events for the  $i^{th}$  bin) and  $\Delta_{pp}$  is the mean peak-to-peak distance, defining the gauge to convert values in ADC channels to number of cells.

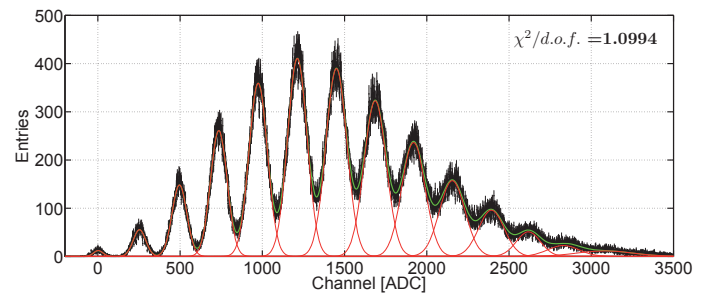


Fig. 8: Outcome of the MGF procedure. Individual Gaussians are in red, while their superposition is displayed in green. The  $\chi^2/d.o.f.$  measures the fit quality.

The value of  $\mu_{MI}$  can be compared to what is estimated presuming a pure Poissonian behaviour and referring to the probability  $P(0)$  of having no fired cell when the expected average value is  $\mu_{ZP}$ , where *ZP* stands for *Zero Peak*:

$$\mu_{ZP} = -\ln(P(0)) = -\ln\left(\frac{A_0}{A_{tot}}\right), \quad (8)$$

being  $A_0$  the area underneath the first peak of the spectrum and  $A_{tot}$  the total number of recorded events. Results are shown in Table IV. The P&P procedure shows a good compatibility with the hypothesis, while the MGF procedure, due to the smaller errors, seems to indicate a discrepancy at the  $3\sigma$  level.

The question can be further investigated considering the full distribution and comparing the experimental probability density function with the assumed model distribution by a  $\chi^2$  test, where:

$$\chi^2 = \sum_{k=0}^{N_{peaks}-1} w_k \times (A_{obs,k} - A_{model,k})^2, \quad (9)$$

being  $A_{obs,k}$  the number of events in the  $k^{th}$  peak of the distribution,  $A_{model,k}$  the corresponding number estimated from the reference model and  $w_k$  the weights accounting for the uncertainties in the content of every bin. Presuming a Poissonian distribution with mean value  $\mu_{MI}$ , the returned values of the  $\chi^2/d.o.f.$  are  $\approx 62$  for the *P&P* procedure and  $\approx 5$  for the *MGF*. The  $\chi^2/d.o.f.$  values, even assuming  $\mu$  as a free parameter, exceeds the 99% C.L. for both methods confirming that the experimental distribution may not be adequately described by a pure Poissonian model.

As a further step, the spectra were compared to the  $P \otimes B$  model introduced in Section III, Eq. 4, where the actual number of fired cells results from avalanches triggered by the incoming photons and by the optical cross-talk. The optimal values of the model parameters, namely the cross-talk probability  $\epsilon_{XT}$  and the mean value  $\mu$  of the distribution of cells fired by photons, are determined by a grid search according to the following iterative procedure [8] :

- the  $\chi^2/d.o.f.$  surface, or  $\Sigma$ , is sliced with planes orthogonal to the  $\epsilon_{XT}$  dimension, at values  $\tilde{\epsilon}_{XT}$  changed with constant step;
- in each slice, the minimum of the  $\Sigma(\tilde{\epsilon}_{XT}, \mu)$  curve is searched and the value  $\mu_{min,0}$  corresponding to the minimum is identified;
- the  $\Sigma(\epsilon_{XT}, \mu_{min,0})$  curve is scanned and the position  $\epsilon_{XT}^*$  of the minimum is identified by a local parabolic fit, to overcome the limitations by the choice of the step in the grid;
- the procedure is repeated for  $\Sigma(\epsilon_{XT}^*, \mu)$  vs  $\mu$ , leading to the determination of the minimum in  $\mu^*$ .

The values  $\mu^*$  and  $\epsilon_{XT}^*$  are retained as estimates of the optimal parameters. Uncertainties are calculated assuming a parabolic shape of the  $\Sigma(\epsilon_{XT}^*, \mu)$  and  $\Sigma(\epsilon_{XT}, \mu^*)$  curves, leading to variances estimated by the inverse of the coefficient

TABLE IV: Estimates of the mean number of fired cells by the average value of the experimental distribution and from the probability of having none, assuming an underlying Poissonian distribution. Errors result from the uncertainties in the peak-to-peak distance and in the area of the zero-cell peak.

	$\mu_{MI}$	$\mu_{ZP}$
P&P	$5.9 \pm 0.2$	$6.1 \pm 0.1$
MGF	$6.24 \pm 0.01$	$6.14 \pm 0.03$

of the quadratic term [8], [9]. The results for the reference spectrum are  $\mu^* = 6.06 \pm 0.01$  and  $\epsilon_{XT}^* = 0.023 \pm 0.002$ .

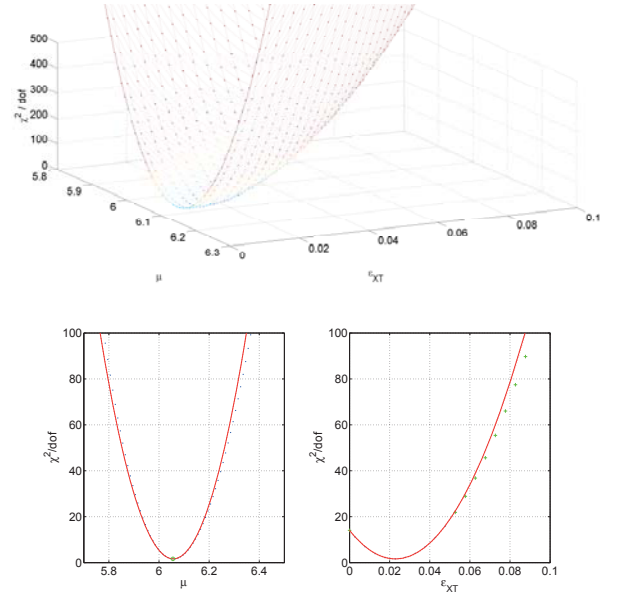


Fig. 9:  $\chi^2/d.o.f.$  surface (top panel) and parabolic behavior nearby its minimum (bottom).

The  $\chi^2/d.o.f.$  curves are shown in Fig. 9, and the result of the fit is displayed in Fig. 10 showing an excellent agreement between data and model.

The quality of the result is confirmed by the data reported in Table V, where the mean value of the Poissonian distribution obtained by the fit ( $\mu^*$ ) and by the *Zero Peak* are compared, together with a comparison between  $\mu_{MI}$  and  $\mu^*(1 + \epsilon_{XT}^*)$ , the mean value of the  $P \otimes B$  distribution.

TABLE V: Estimates of the mean number of fired cells for pure Poissonian and  $P \otimes B$  models.

	$\mu^*$	$\mu_{ZP}$
Mean Value of the Poissonian distribution	$6.06 \pm 0.01$	$6.14 \pm 0.03$
	$\mu_{MI}$	$\mu^*(1 + \epsilon_{XT}^*)$
Mean Number of Fired Cells	$6.24 \pm 0.01$	$6.20 \pm 0.01$

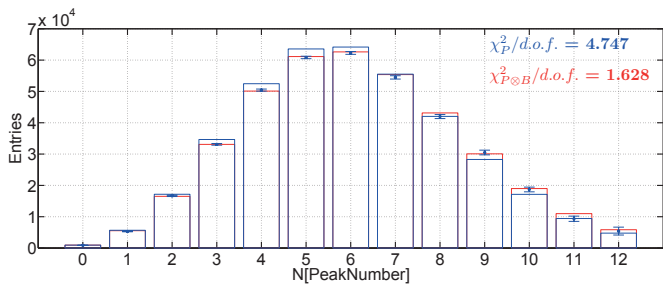


Fig. 10: Data from the reference spectrum are compared to a simple Poissonian model (blue) and to the  $P \otimes B$  model (red), accounting for the optical crosstalk. The  $\chi^2$  value rule out the former at 99% C.L..

Results by the other recorded spectra are summarized in Table VI and Fig. 11 for the MGF procedure, confirming the validity of the  $P \otimes B$  model and the need to account for detector effects to have a proper understanding of the phenomenon being investigated.

TABLE VI: Estimate of the mean number of fired cells with the  $P \otimes B$  model using the RUN1 and RUN2 data-sets. Also in this case, the  $P \otimes B$  model shows an agreement at the 99% C.L.. The measured  $\chi^2$  is 13.34 for the RUN1 and 14.27 for the RUN2 respectively.

	$\mu^*$	$\mu_{ZP}$
Mean Value of the Poissonian distribution	$1.133 \pm 0.004$	$1.143 \pm 0.002$
	$1.657 \pm 0.004$	$1.667 \pm 0.003$
	$\mu_{MI}$	$\mu^*(1 + \epsilon_{XT}^*)$
Mean Number of Fired Cells	$1.209 \pm 0.002$	$1.220 \pm 0.01$
	$1.762 \pm 0.002$	$1.777 \pm 0.01$

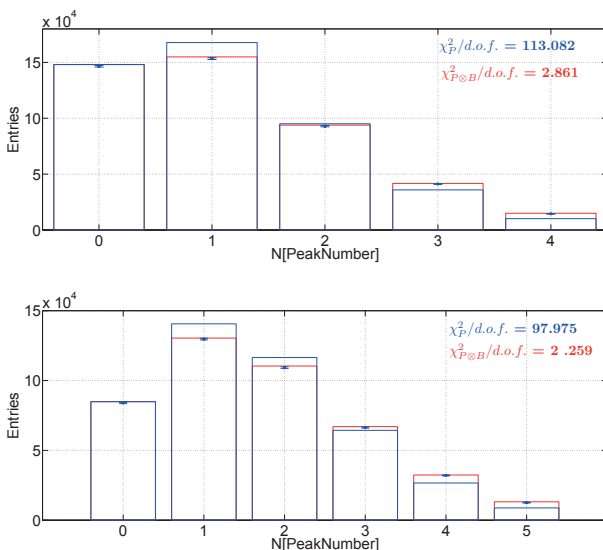


Fig. 11: Results of the MGF procedure on the low and middle intensity, RUN1 and RUN2.

## VI. CONCLUSION

Instruments and methods for the investigation of the statistical properties of the light emitted by an incoherent source have been proposed, based on Silicon Photomultipliers embedded into a flexible, modular, easy-to-use kit.

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